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Newtonian gravitons and D-brane collective coordinates in wound string theory

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ABSTRACT: Recently it was shown that NCOS theories are part of a ten-dimensional theory known as Non-relativistic Wound string theory. We clarify the sense in which gravity is present in this theory. We show that Wound string theory contains exceptional unwound strings, including a graviton, which mediate the previously discovered instantaneous long-range interactions, but are negligible as asymptotic states. Unwound strings also provide the expected collective coordinates for the transverse D-branes in the theory. These and other results are shown to follow both from a direct analysis of the effect of the NCOS limit on the parent string theory, and from the worldsheet formalism developed by Gomis and Ooguri, about which we make some additional remarks. We also devote some attention to supergravity duals, and in particular show that the open and closed strings of the theory are respectively described by short and long strings on the supergravity side.

KEYWORDS: Superstrings and Heterotic Strings, D-branes, M(atrix) Theories, AdS-CFT Correspondance.

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1. Introduction

The emergence of noncommutativity in string theory [1, 2, 3] has recently been brought into sharper focus through the discovery of Noncommutative Open String (NCOS) [4, 5, 6] and Open Brane (OM/ODp) [6, 7, 8] theories. These theories have generated interest not only because they display a certain form of noncommutativity between space and time, but also because they stand midway between field theories and conventional string/M-theory, since their fluctuation spectrum is believed not to include gravitons.

NCOS theory in $p+1$ dimensions ($p \leq 5$) is obtained by considering a Dp-brane (or a stack of them) in a low-energy limit $l_s \to 0$ where the electric field on the brane is made to approach its critical value [4, 5]. These theories were initially believed to contain only open strings, which in particular means they do not include gravity. In an important paper, Klebanov and Maldacena [9] discovered that, when the direction of the electric field is compactified on a circle of radius R, closed strings with strictly positive winding number are also present in the theory. The graviton ($w = 0$) was still thought to be absent, even though the spectrum does include its $w > 0$ cousins.

Since (for finite R) closed strings can leave the brane, one is actually dealing with a D-dimensional theory ($D = 10$ for the superstring). It is then natural to wonder whether the theory makes sense even in the absence of the Dp-brane(s), and recently this question has been answered affirmatively [10, 11]. As explained in those works, starting with any one of the conventional string theories, it is possible to single out a spatial direction (we will take it to be x^1, and refer to it as the longitudinal direction), compactify it on a circle of radius R, and consider a limit where the coupling constant, string length, and (closed string) metric scale as

$$g_s = \frac{G_s}{\sqrt{\delta}} \to \infty\,, \quad l_s = L_s\sqrt{\delta} \to 0\,, \quad g_{\mu\nu} = (-1, 1, \delta, \delta, \ldots)\,, \text{ with } G_s, L_s, R \text{ fixed}\,. \tag{1.1}$$

The result is a consistent D-dimensional theory characterized by the fact that all objects in it must carry strictly positive F-string winding along the longitudinal direction, and consequently designated Wound String theory in [10]. This theory was discovered independently by Gomis and Ooguri [11], who chose to refer to it as Non-relativistic (or galilean) Closed String theory, to emphasize the fact that its closed string spectrum has a non-relativistic form. The parameters G_s and L_s introduced in (1.1) are the effective coupling constant and string length of the theory.[1]

States without any D-branes contain of course only closed strings (with $w > 0$), but if there are D-branes in the spectrum of the parent string theory, the same will be true for the corresponding Wound string theory. To comply with the $w > 0$ requirement, a Dp-brane extended along the longitudinal direction must carry a near-critical electric field, thus giving rise to a setup which is exactly what is known as NCOS theory[2] [10, 11]. Wound string theory contains also transverse D-branes [10].

Wound string theory is defined with a built-in compactification, but one may of course consider the decompactification limit $R \to \infty$ as a special case. In this limit the wound strings decouple from the worldvolume theory on longitudinal D-branes, so the latter becomes the original NCOS theory of [4, 5], which was defined on a non-compact space. Note that this decoupling occurs not because the energy of the wound strings diverges in the limit (the D-branes carry F-string winding, so their energy diverges at the same rate), but because the energy cost for a D-brane to emit a closed string into the bulk (i.e., the binding energy) is proportional to R. In fact, when the theory is examined at finite temperature, wound strings are seen to play an important role even as $R \to \infty$. Indeed, it was shown in [12] that the Hagedorn

[1] As was stressed in [10], the coupling constant of the theory is really the dimensionful combination $G_sL_s = g_sl_s$. The relation between G_s, L_s and the NCOS parameters G_o^2, α'_e is discussed in [10] and in section 3.2.1 of the present paper.

[2] Note that the NCOS name would not be appropriate for the full theory, not only because it contains closed strings, but also because space/time noncommutativity is not an intrinsic feature of the theory as a whole (e.g., there is no sign of it in closed string scattering amplitudes [10, 11]). Noncommutativity is a property only of the worldvolume of longitudinal D-branes in this theory.

transition in NCOS theories occurs when the temperature is large enough that the entropic contribution of these strings to the free energy becomes larger than their binding energy, resulting in their liberation from the D-brane.

As shown explicitly in [10, 11] (see also [13]), longitudinal T-duality converts the limit (1.1) into the limit of discrete light-cone quantization (DLCQ) in the sense of [14, 15, 16], so Wound string theory is T-dual to DLCQ string theory.[3] It is also known to be U-dual to various Wrapped (galilean) Brane theories [10, 11], which contain the OM/ODp/NCYM theories as special classes of states [10]. A Wrapped Xp-brane theory is defined by starting with a parent string/M-theory compactified[4] on a p-cycle, and then taking a limit analogous to (1.1), which truncates the spectrum down to those objects carrying strictly positive Xp-brane wrapping number on the p-cycle in question. Because of their connection to DLCQ and Matrix theory [17, 18] on the one hand, and to the theories with noncommutativity on the other, the Wound/Wrapped theories offer a novel and unified perspective on several recurrent themes of recent years, and could thus facilitate new developments. With this motivation in mind, we will continue here with the study of Wound string theory, making use of the two complementary approaches developed in [10] and [11].

The central theme of this paper will be the presence of unwound closed strings in the theory, which are thus exceptions to the general $w > 0$ rule. This exception was noted already in [10], and will be elaborated upon in section 2. Strings with $w = 0$ can survive the limit (1.1) only if they carry vanishing transverse momentum and have zero oscillator number; the graviton is among such exceptional states. Due to the $p_\perp = 0$ restriction, the unwound strings constitute a zero-measure set in phase space, and were therefore believed in [10] not to play any dynamical role in the theory.

As will be explained in sections 2.1 and 2.2, even though their exceptional character indeed implies that they can be ignored as asymptotic states, the unwound strings do play one role: as the only massless objects in the theory, they are responsible for the newtonian long-range interactions discovered in [11]. Whereas on-shell these strings cannot carry any transverse momentum (for otherwise their energy would diverge), off-shell their transverse momentum is unrestricted, allowing them to act as mediators of a long-range force. The metric rescaling in (1.1) effectively takes the transverse speed of light to infinity, so this force is transmitted instantaneously. Wound string theory is thus seen to contain newtonian gravitons.

Section 2.3 shows that this same conclusion follows from the worldsheet formalism of [11], and some additional remarks are made there regarding that formalism.

[3]Notice this implies that the limit $R \to \infty$ looks rather exotic from the DLCQ perspective: it corresponds to shrinking the radius of the null circle to zero size.

[4]The fact that the compactification need not be toroidal was first pointed out in [11]. A simple but interesting example is M-theory compactified on $S^1 \times S^1/\mathbb{Z}_2$ — this yields a Wrapped M2-brane theory which is the eleven-dimensional lift of Wound Heterotic $E_8 \times E_8$ string theory.

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In particular, we note the existence of a free parameter which can be adjusted to simplify the form of the bosonic action, and we write down the fermionic action.

In section 3 we turn our attention to D-branes, where the exceptional unwound strings find a second role: open strings with $w = 0$ and oscillator level $N = 0$ give rise to the expected collective coordinates for transverse D-branes (as well as to newtonian photons on the brane worldvolume). This is explained in section 3.1, where the excitation spectrum for transverse D-branes is shown to be of the same form as the closed string spectrum. In section 3.2 this spectrum is reproduced using the methods of [11], after having reviewed the results of that paper for longitudinal D-branes, in order to clarify their dependence on the parameters of the theory.

Section 4 explains how some of the above properties of Wound string theory can be understood from the point of view of the known supergravity backgrounds dual to the longitudinal Dp-branes of the theory (i.e., NCOS theory) [5, 19]. In particular, the point is made that the supergravity dual describes not only the open strings attached to the branes, but also the closed $w > 0$ strings that are free to move off of them.

Our conclusions are summarized in section 5, where we include additional comments on the sense in which gravity is present in the theory, and outline some of the directions for future work.

2. The physics of unwound strings

2.1 Long-range interactions from unwound strings

As explained in [10, 11], all objects in Wound string theory carry strictly positive F-string winding, and are consequently massive. We would therefore expect interactions to be short-ranged, giving rise to potentials which decrease exponentially with distance. It was discovered in [11] that this is actually not the case. To understand why, consider a process where two wound strings scatter off each other in the parent string theory (i.e., before taking the limit). For simplicity we will (as in [11]) focus on 'tachyons' (with positive winding number) in bosonic string theory. For tachyons, the level matching condition $N_L - N_R = nw$ implies that $n = 0$ for all four vertices. The scattering amplitude is then found to take the same form as the one for unwound tachyons; this agreement is a trivial consequence of T-duality. In particular, at tree level one obtains the familiar Virasoro-Shapiro amplitude (see, e.g., [20]), including a factor

$$\Gamma\left(-1 - \frac{t}{4} - \frac{R^2}{4\alpha'}(w_1 + w_3)^2\right), \tag{2.1}$$

where $t = -\alpha'(p_1 + p_3)^2$. One thus finds the expected t-channel poles at energies such that $t = 4N - (w_1 + w_3)^2 R^2/\alpha'$ (where $N = -1, 0, 1, \ldots$), corresponding to the usual spectrum for string theory on a circle. For the purpose of extracting a potential, we restrict attention to the case of zero winding transfer, $w_3 = -w_1$. The poles are then at $t = -4, 0, 4, \ldots$ In the limit (1.1) we scale $\alpha' \to 0$, so all $t \neq 0$

poles move off to infinite energy. The $t = 0$ pole suffers the same fate, *unless* there is zero perpendicular-momentum transfer, $(\vec{k}_1 + \vec{k}_3)_\perp = 0$ (where $\vec{k}_\perp$ is the transverse momentum in the coordinates where the metric scales as in (1.1)). Close to this pole the amplitude is proportional to $(\vec{k}_1 + \vec{k}_3)_\perp^{-2}$, which upon Fourier transformation gives rise to a newtonian potential [11].

This result is general: as discussed in [10], tree-level closed string scattering amplitudes in Wound string theory have the same form as those in the parent theory, with the kinematic variables s, t, u taking their limiting values.[5] In all processes one thus finds a t-channel pole which remains at finite energy even when no winding number is exchanged. The presence of this pole leads us to conclude that the spectrum of the Wound theory includes exceptional closed string states with zero winding number. As seen above, these states must carry vanishing transverse momentum, and are forced to have oscillator levels $N_L = N_R = 0$ (so they include the graviton). That these states remain in the theory was noted already in [10], although their role as carriers of a newtonian force was not recognized. Starting from the usual energy formula for a closed string,

$$p_0 = \sqrt{\left(\frac{wR}{\alpha'}\right)^2 + \left(\frac{n}{R}\right)^2 + \frac{|\vec{k}_\perp|^2}{\delta} + \frac{2}{\alpha'}(N_L + N_R)}\,, \tag{2.2}$$

it is seen that in the limit (1.1) a state with $w = 0$ has a diverging energy $p_0 \propto \delta^{-1/2}$, and is therefore removed from the spectrum *unless* $\vec{k}_\perp = N_L = N_R = 0$, in which case $p_0 = |n|/R$.

Wound string theory is thus seen to contain gravitons, dilatons, and antisymmetricons (plus massless Ramond-Ramond and fermionic states, if these were present in the parent theory). On-shell, these unwound strings are forced to have vanishing transverse momenta, so they constitute a negligible zero-measure set in the space of asymptotic states (we will return to this point in the next subsection). Even so, the fact that they are the only massless ($w = 0$) states in the theory implies that, off-shell, they act as the sole mediators of long-range interactions.

Notice that the fact that these strings are unwound implies that their energy is finite even in the decompactification limit $R \to \infty$. This means in particular that they should show up in the original NCOS one-loop scattering amplitudes. The nonplanar annulus amplitude for $2 \to 2$ open string scattering was considered in [5], and indeed, it can be seen from the equation following (4.4) on that paper that if $k_\perp = N = 0$ then there is a 'pole' at finite $p_0 = |p_1|$. As explained there, the pole appears integrated in momentum-space over the directions transverse to the Dp-brane, and for $p < 7$ there is no real singularity. The physical reason for this is that the brane, understood as an object which is completely localized along the directions transverse to it, cannot emit a closed string with a definite transverse momentum (in

[5]For some other amplitude calculations in Wound string theory see [5, 9, 21, 22].

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this case, $k_\perp = 0$). This conclusion was confirmed in [12], where it was shown that the cross-section for graviton production by the brane vanishes for all $p < 7$. The statement that NCOS theories are decoupled from gravity [4, 5, 6, 9, 12] is therefore seen to be related to the zero-measure character of the newtonian gravitons (see the next subsection). At the same time, it is clear that, whether R is finite or not, the long-range interactions between D-branes (just like those between unwound strings) are controlled by off-shell strings with $w = 0$.

The properties of the unwound strings also follow on very general grounds from the nature of the limit (1.1). More specifically, note that the rescaling of the transverse metric implies that the relevant physics in the theory takes place on very small transverse scales. As a consequence, the effective speed of light is taken to infinity, $c = \sqrt{|g_{\perp\perp}/g_{00}|} \sim 1/\sqrt{\delta}$, and physics becomes non-relativistic. This is consistent with the limit (1.1) being equivalent (T-dual) to the DLCQ limit, as was shown explicitly in [10, 11] (see also [13]). If we then consider the fate of a massless on-shell particle, obeying $E = ck$, as the speed of light is taken to infinity, we find that finite energy indeed requires vanishing momentum in the limit.

As we have seen above, the unwound strings are responsible for the instantaneous gravitational force between the various objects of the theory. The force is instantaneous because the speed of light is infinite. The carriers of the force are, as usual, off-shell, which for finite E and finite k actually means that the particles must be infinitely off-shell, i.e., $\Delta E \sim kc \sim 1/\sqrt{\delta}$. From the uncertainty principle, the particles can then exist only for a vanishingly short time $\Delta t \sim \hbar/\Delta E \sim \sqrt{\delta}$, which nevertheless allows them to reach any finite distance $s \sim c\Delta t \sim \hbar/k$.

Note that strings with $w = 0$ are related by longitudinal T-duality to DLCQ strings with $p_- = 0$. In the conventional treatment of DLCQ field theories [23, 24], states with vanishing longitudinal momentum are not independent degrees of freedom; they satisfy a constraint which can in principle be used to eliminate them from the theory — this is the infamous zero-mode problem (for recent progress, see [25]). When DLCQ is defined instead as a limit of compactification on a small spatial circle [14, 15], as it is in our case, the treatment of zero modes changes. Their effect for *field* theories was analyzed in [15], where they were shown to become strongly-coupled and thus complicate the analysis of the DLCQ limit (see also [26]). The situation in string/M-theory appears to be more benign: unlike their field theory counterparts, perturbative string scattering amplitudes are well-defined in the DLCQ limit [27, 28, 29]. Through T-duality, this is true also for amplitudes in Wound string theory [10, 11].

2.2 Unwound strings as asymptotic states

We would now like to show that unwound strings are physically irrelevant as asymptotic states: measurable quantities are always extracted from scattering amplitudes by integrating over a phase space in which unwound strings constitute a zero-measure set. To make this intuitive argument more precise, consider as a concrete example the process in the parent string theory through which a wound string with energy, momentum and winding $(E, \vec{p}, n, w)$ decays into two strings, labelled 1 and 2, for which these quantities take the values $(E_i, \vec{p}_i, n_i, w_i)$, $i = 1, 2$. The decay rate takes the usual form

$$\Gamma \propto \frac{1}{R} \sum_{n_2} \int \frac{d\vec{p}_2}{(E - E_2)E_2} \delta(E - E_1 - E_2) \mathcal{A}(p, p_2)\,, \tag{2.3}$$

where the phase space integration is only over possible states for string 2; the state of string 1 is determined by the conservation laws $\vec{p}_1 = \vec{p} - \vec{p}_2$, $n_1 = n - n_2$, $w_1 = w - w_2$. The three-point amplitude $\mathcal{A}(p, p_2)$ is just a vertex involving the momenta and polarization tensors for the three strings. Without loss of generality, we can work in the reference frame where the decaying particle is at rest in the transverse directions, $\vec{p} = 0$.

Now, in terms of coordinates where the metric of the parent theory scales as in (1.1), the coordinate momenta $\vec{k}_{1,2}$ are related to the proper transverse momenta $\vec{p}_{1,2}$ through $\vec{p} = \vec{k}/\sqrt{\delta}$. For any given value of δ, we can use $\vec{k}_2$ instead of $\vec{p}_2$ as the integration variable in (2.3). The jacobian for this is $\delta^{(D-2)/2}$, where D is the total spacetime dimension. Since the transition rate is given as the number of transitions per unit volume and time, this jacobian is precisely what is needed in order to obtain a finite rate per unit coordinate volume, and we will therefore not write it explicitly.

Consider first the case when string 2 is unwound: $w_2 = N_{L,2} = N_{R,2} = 0$. Recalling the energy formula (2.2) we see that

$$E_2 = \sqrt{\left(\frac{n}{R}\right)^2 + \frac{|\vec{k}_2|^2}{\delta}} \tag{2.4}$$

and

$$E - E_1 = -\frac{{L_s}^2}{2wR} |\vec{k}_2|^2 + \frac{N - N_1}{wR} - \frac{|\vec{k}_2|}{\sqrt{\delta}} + \mathcal{O}(\delta)\,, \tag{2.5}$$

where we have let $N = N_L + N_R$. The delta-function in (2.3) thus forces

$$|\vec{k}_2| = \frac{(N - N_1)}{wR} \sqrt{\delta} + \mathcal{O}(\delta^{3/2})\,, \tag{2.6}$$

which through (2.4) implies that $E_2 \sim \mathcal{O}(1)$. Using the delta-function to dispose of the radial integral over $|\vec{k}_2|$ in (2.3), we conclude that

$$\Gamma_{w=0} \propto \delta^{(D-2)/2}\ \mathcal{A}|_{|\vec{k}_2| \sim \mathcal{O}(\sqrt{\delta})} \tag{2.7}$$

Consider now the case when string 2 is wound, $w_2 > 0$. It is easy to see that the energy delta-function in (2.3) then constrains $|\vec{k}_2| \sim \mathcal{O}(1)$, and therefore

$$\Gamma_{w>0} \propto \mathcal{A}|_{|\vec{k}_2| \sim \mathcal{O}(1)} \tag{2.8}$$

In both cases $\mathcal{A}$ is a function of the transverse momenta $\vec{p} = \vec{k}/\sqrt{\delta}$ and the (finite) energies of the strings, which so we conclude that in the limit of interest $\Gamma_{w=0}$ is vanishingly small compared to $\Gamma_{w>0}$.

A nice way to understand the result is to make an analogy with bremsstrahlung. The energy loss for an accelerated charged particle is given by

$$\frac{dW}{dt} \sim \frac{Q^2 a^2}{c^3},$$

where a is the acceleration and c is the speed of light. Q^2 (the charge squared) is defined as having dimensions of energy times length, so that the potential energy for two charges is of the form

$$V = \frac{Q_1 Q_2}{r}.$$

We should now consider the non-relativistic limit, with the speed of light going to infinity. While doing that we need to make sure that Q is finite in order for the strength of the electric forces to remain unaffected. In this limit the bremsstrahlung goes away, showing that the energy loss is a relativistic effect. This is of course connected with the fact that fields are relativistic constructions. It is only in relativity, with a finite speed of light, where fields are needed in order to carry a force — fields which also might serve as an energy dump as in the case of bremsstrahlung. In a direct-action theory there is never any asymptotic state associated with a carrier of the force, since there is always a recipient at the other end of the line. As we have seen, the story is completely analogous in the case of the unwound strings.

2.3 Worldsheet perspective

Wound string theory is defined as the specific limit (1.1) of a standard string theory, so all of its properties can be deduced by focusing on the corresponding aspect of the parent theory and studying the effect of the limit. This is the approach adopted in [4, 5, 9, 10] and in the preceding subsections of this paper. A complementary approach, developed by Gomis and Ooguri [11], is to take the limit once and for all at the level of the worldsheet action. This has the advantage of producing a finite worldsheet lagrangian which serves as a more explicit definition of the theory [11]. Also, for actual calculations, using the resulting worldsheet rules will in general be more convenient than taking the limit in each case separately. On the other hand, in this approach the relation to the parent theory is not transparent, and it is therefore important to make sure that the formalism correctly captures all properties of the theory. We will now examine this question in relation to the unwound strings discussed in the previous subsections, which, in fact, did not appear in the analysis of [11]. We will find that the treatment of $w = 0$ strings requires special care. Along the way we will make some additional remarks regarding the Gomis-Ooguri formalism.

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In the approach of [11], the bosonic part of the usual string action in the presence of a B_{01}-field is first rewritten in the form[6]

$$S=\int\frac{d^2z}{2\pi}\left\{\beta\bar{\partial}\gamma+\tilde{\beta}\partial\tilde{\gamma}-\frac{2(l_s/L_s)^2}{1+B}\beta\tilde{\beta}+\frac{1-B}{2(l_s/L_s)^2}\partial\gamma\bar{\partial}\tilde{\gamma}+\frac{1}{{L_s}^2}\partial X^i\bar{\partial}X^i\right\}$$

where $i=2,\ldots,D-1$,

$$L_s\gamma\equiv X^+\equiv X^0+X^1,\qquad L_s\tilde{\gamma}\equiv X^-\equiv -X^0+X^1, \tag{2.9}$$

and $\beta,\tilde{\beta}$ are Lagrange multipliers. The limit (1.1) is then taken while simultaneously making the B-field approach its critical value according to

$$B\equiv B_{01}=1-\lambda\left(\frac{l_s}{L_s}\right)^2, \tag{2.10}$$

where we have included a free parameter λ. Keeping the leading and subleading terms in $(l_s/L_s)^2$, the result is

$$S=\int\frac{d^2z}{2\pi}\left\{\beta\bar{\partial}\gamma+\tilde{\beta}\partial\tilde{\gamma}-\left(\frac{l_s}{L_s}\right)^2\beta\tilde{\beta}+\frac{\lambda}{2}\partial\gamma\bar{\partial}\tilde{\gamma}+\frac{1}{{L_s}^2}\partial X^i\bar{\partial}X^i\right\}. \tag{2.11}$$

Gomis and Ooguri [11] then proceeded by putting $(l_s/L_s)^2=0$ identically, thereby arriving at

$$S=\int\frac{d^2z}{2\pi}\left\{\beta\bar{\partial}\gamma+\tilde{\beta}\partial\tilde{\gamma}+\frac{\lambda}{2}\partial\gamma\bar{\partial}\tilde{\gamma}+\frac{1}{L_s^2}\partial X^i\bar{\partial}X^i\right\}. \tag{2.12}$$

The Lagrange multipliers then constrain X^+ and X^- to be respectively analytic and antianalytic.[7] As we will argue, however, dropping the $\beta\tilde{\beta}$ term in (2.11) is strictly speaking correct only if $w\neq 0$.

Let us now explain the significance of the parameter λ. As explained in [10], as far as closed strings are concerned, the role of the B-field is merely to remove a divergent contribution to the energy arising from the winding term $|w|R/\alpha'$. As always, when subtracting this infinity one has the option of leaving behind a finite term, and this freedom is parametrized by λ. The simplest choice is $\lambda=0$, which corresponds to not leaving any such term behind [10]. The β-γ action (2.12) is then exactly that of a system of commuting ghosts. For open strings associated with longitudinal D-branes the situation is more subtle. At first sight, it would seem like $\lambda=0$ is in that case not allowed; the authors of [11] chose instead $\lambda=1/2$, to conform with the usual NCOS convention [4, 5, 9]. In section 3.2.1 we will amplify the discussion of [10] in this regard, emphasizing that the NCOS conventions are no longer useful now that they are understood to refer only to a particular subsector of

[6]In comparing with [11], note that $B_{\rm here}=2\pi\alpha' B_{\rm there}$.
[7]That this is the effect of the limit was pointed out already in [4].

the full Wound string theory. It will become clear there that the open string spectrum is in fact independent of λ, so we can consistently set $\lambda = 0$. In the meantime λ is left arbitrary to keep track of its effect on the expressions to follow.

Before proceeding with the review of [11], and the extension of their results to $w = 0$, let us note that the fermionic part of the action can be easily put into a form analogous to the β-γ system (2.12). In the presence of a background B-field, the action for the left-moving fermions reads [30]

$$S_\psi = \int \frac{d^2z}{4\pi}(g_{\mu\nu} + B_{\nu\mu})\psi^\mu\bar{\partial}\psi^\nu\,. \tag{2.13}$$

For the transverse part of the action, the property $\int d\theta \exp(-A\theta) = A\int d\theta \exp(-\theta)$ can be used to bring $g_{ii} = (l_s/L_s)^2 \to 0$ out of the fermionic path integrals, where it can be absorbed through a rescaling of the overall normalization factor. The limit (1.1) + (2.10) can then be taken without any difficulty, yielding

$$S_\psi = \int \frac{d^2z}{2\pi}\left\{b\bar{\partial}c + \frac{1}{2}\psi^i\bar{\partial}\psi^i\right\}\,, \tag{2.14}$$

where we have defined $b = (-\psi^0 + \psi^1)/\sqrt{2}$, $c = (\psi^0 + \psi^1)/\sqrt{2}$. So, just like $X^{0,1}$ reduce in the limit to a system of commuting ghosts (with conformal weights $h_\beta = 1$, $h_\gamma = 0$) [11], we see that $\psi^{0,1}$ are equivalent to a system of anticommuting ghosts (with weights $h_b = h_c = 1/2$). For simplicity, in the rest of the paper we will concentrate on the bosonic part of the system.

Let us now proceed with the general analysis by writing down the equations of motion:

$$\begin{aligned}
\bar{\partial}\gamma &= \left(\frac{l_s}{L_s}\right)^2\tilde{\beta}, \qquad \bar{\partial}\beta + \frac{\lambda}{2}\partial\bar{\partial}\tilde{\gamma} = 0\,,\\
\partial\tilde{\gamma} &= \left(\frac{l_s}{L_s}\right)^2\beta, \qquad \partial\tilde{\beta} + \frac{\lambda}{2}\partial\bar{\partial}\gamma = 0\,,
\end{aligned} \tag{2.15}$$

Note how the presence of the $\beta\tilde{\beta}$-term provides antianalytic contributions to γ, and analytic contributions to $\tilde{\gamma}$. The mode expansions are

$$\beta(z) = \sum_{n=-\infty}^{\infty}\beta_n z^{-n-1}\,, \qquad \tilde{\beta}(\bar{z}) = \sum_{n=-\infty}^{\infty}\tilde{\beta}_n\bar{z}^{-n-1}\,, \tag{2.16}$$

$$\gamma(z,\bar{z}) = \left[+i\left(\frac{wR}{L_s}\right) + \frac{l_s^2}{L_s^2}\tilde{\beta}_0\right]\log z + \sum_{n=-\infty}^{\infty}\gamma_n z^{-n} + \frac{l_s^2}{L_s^2}\tilde{\beta}_0\log\bar{z} - \frac{l_s^2}{L_s^2}\sum_{n\neq 0}\frac{\tilde{\beta}_n}{n}\bar{z}^{-n}\,,$$

$$\tilde{\gamma}(z,\bar{z}) = \left[-i\left(\frac{wR}{L_s}\right) + \frac{l_s^2}{L_s^2}\beta_0\right]\log\bar{z} + \sum_{n=-\infty}^{\infty}\tilde{\gamma}_n\bar{z}^{-n} + \frac{l_s^2}{L_s^2}\beta_0\log z - \frac{l_s^2}{L_s^2}\sum_{n\neq 0}\frac{\beta_n}{n}z^{-n}\,.$$

where R is the radius of the longitudinal direction. (If this direction is not compact we must restrict attention to the states with $w = 0$.) The OPE's imply that the only non-zero commutators are

$$[\gamma_n, \beta_m] = \delta_{n+m}\,, \qquad [\tilde{\gamma}_n, \tilde{\beta}_m] = \delta_{n+m}\,. \tag{2.17}$$

The contribution from the directions 01 ('parallel' to B) to the energy-momentum tensor is

$$T^{||}(z) = -:\beta\partial\gamma:\,, \qquad T^{||}(\bar{z}) = -:\tilde{\beta}\bar{\partial}\tilde{\gamma}:\,, \tag{2.18}$$

and the corresponding Virasoro modes are

$$\begin{aligned} L_n^{||} &= \left[-i\left(\frac{wR}{L_s}\right) - \left(\frac{l_s}{L_s}\right)^2 \tilde{\beta}_0\right]\beta_n + \sum_m m:\beta_{n-m}\gamma_m:\,, \\ \tilde{L}_n^{||} &= \left[+i\left(\frac{wR}{L_s}\right) - \left(\frac{l_s}{L_s}\right)^2 \tilde{\beta}_0\right]\tilde{\beta}_n + \sum_m m:\tilde{\beta}_{n-m}\tilde{\gamma}_m:\,. \end{aligned} \tag{2.19}$$

Assume for now that $w > 0$. Terms of order $(l_s/L_s)^2$ are then subleading in all expressions, and can therefore be dropped, as was done in [11]. One then has, in particular,

$$L_0^{||} = -i\beta_0\left(\frac{wR}{L_s}\right) + N_{||}\,, \qquad \tilde{L}_0^{||} = +i\tilde{\beta}_0\left(\frac{wR}{L_s}\right) + \tilde{N}_{||}\,. \tag{2.20}$$

From (2.12), the momenta conjugate to γ and $\tilde{\gamma}$ are

$$\begin{aligned} \Pi_\gamma &\equiv i\frac{\partial\mathcal{L}}{\partial\dot{\gamma}} = L_s\Pi_+ = \frac{i}{2\pi}\left(z\beta + \frac{\lambda}{2}\bar{z}\bar{\partial}\tilde{\gamma}\right)\,, \\ \Pi_{\tilde{\gamma}} &\equiv i\frac{\partial\mathcal{L}}{\partial\dot{\tilde{\gamma}}} = L_s\Pi_- = \frac{i}{2\pi}\left(\bar{z}\tilde{\beta} + \frac{\lambda}{2}z\partial\gamma\right)\,. \end{aligned} \tag{2.21}$$

(The dot denotes differentiation with respect to $\sigma_2 = \ln|z|$.) The zero mode piece of these equations reads

$$\begin{aligned} L_s p_+ &\equiv L_s\frac{1}{2}(+p_0 + p_1) = i\beta_0 + \frac{\lambda}{2}\left(\frac{wR}{L_s}\right)\,, \\ L_s p_- &\equiv L_s\frac{1}{2}(-p_0 + p_1) = i\tilde{\beta}_0 - \frac{\lambda}{2}\left(\frac{wR}{L_s}\right)\,, \end{aligned} \tag{2.22}$$

from which it follows that [11]

$$\begin{aligned} p_0 &= \frac{i(\beta_0 - \tilde{\beta}_0)}{L_s} + \lambda\left(\frac{wR}{{L_s}^2}\right)\,, \\ p_1 &= \frac{i(\beta_0 + \tilde{\beta}_0)}{L_s}\,. \end{aligned} \tag{2.23}$$

If the longitudinal direction is compact, then of course $p_1 = n/R$.

Let us now determine the spectrum. The perpendicular directions give rise to the usual Virasoro modes, including

$$L_0^{\perp} = \frac{{L_s}^2}{4} p_{\perp}^2 + N_{\perp}, \qquad \tilde{L}_0^{\perp} = \frac{{L_s}^2}{4} p_{\perp}^2 + \tilde{N}_{\perp}\,. \tag{2.24}$$

For convenience, we use transverse number operators $N_{\perp}, \tilde{N}_{\perp}$ which are shifted by -1 with respect to the usual definition,[8] so that the physical state conditions read

$$L_n \equiv L_n^{||} + L_n^{\perp} = 0\,, \qquad \tilde{L}_n \equiv \tilde{L}_n^{||} + \tilde{L}_n^{\perp} = 0 \qquad \text{for all} \quad n \geq 0\,. \tag{2.25}$$

The possible eigenvalues of $N_{\perp}, \tilde{N}_{\perp}$ are then $-1, 0, 1, \ldots$ The level-matching condition $L_0 = \tilde{L}_0$ implies[9]

$$N_{||} + N_{\perp} - \tilde{N}_{||} - \tilde{N}_{\perp} = nw\,. \tag{2.26}$$

The spectrum follows from the requirement $L_0 = \tilde{L}_0 = 0$. As shown in [11], through (2.20), (2.23), and (2.24) these two conditions imply that for $w \neq 0$,

$$p_0 = \lambda \frac{wR}{{L_s}^2} + \frac{{L_s}^2 p_{\perp}^2}{2wR} + \frac{N + \tilde{N}}{wR}\,, \tag{2.27}$$

where $N = N_{||} + N_{\perp}$, $\tilde{N} = \tilde{N}_{||} + \tilde{N}_{\perp}$. This spectrum was originally derived in [9]. Incidentally, notice that the finite negative energy assigned by (2.27) to a state with $w < 0$ is an artifact of the formalism; in reality such states have energies which are positive and infinitely larger than those of the positively wound states [10].

Having reviewed the results of [11], let us now consider the unwound states, $w = 0$. We focus on the left-movers, but the story is analogous for the right-movers. Using (2.20) and (2.24), we learn that in order to satisfy $L_0 = \tilde{L}_0 = 0$ we must demand that $(L_s p_{\perp})^2 = -4N = -4\tilde{N}$. Other than the tachyon, the only possibility is $p_{\perp} = N = \tilde{N} = 0$, i.e., 'gravitons' with zero perpendicular momentum. These are precisely the unwound strings discussed in the previous subsections. *A priori*, they can be polarized along the transverse directions ($N_{||} = 0$, $N_{\perp} = 0$) or along the parallel directions ($N_{||} = 1$, $N_{\perp} = -1$). To ascertain this we must remember to enforce (2.25) not only for $n = 0$, but also for all $n > 0$. The expectation would then be that any time-like polarization would be removed by the constraints, while a longitudinal polarization would be gauge. Unfortunately, one discovers that the L_1 constraint, as given by (2.19) with $w = 0$ and $(l_s/L_s)^2 = 0$, requires only that $\beta_0\gamma_1 = 0$. If we in addition have $\beta_0 = 0$ (i.e., $p_- = 0$ but in general $p_+ \neq 0$), the constraint fails to remove both the γ_{-1} and the β_{-1} states. Furthermore, since the $\beta_0\gamma_{-1}$ term in L_{-1} similarly disappears, neither the γ_{-1} nor the β_{-1} are spurious for $\beta_0 = 0$. The result is therefore a situation where all polarizations are physical, and there are undesired negative-norm states.

[8] The shift by -1 refers to the bosonic string. For the superstring we would shift by $-1/2$ in the NS sector, and by 0 in the R sector.

[9] Due to our conventions, for the NS-R sector of the superstrings there would actually be an additional constant in (2.26).

However, it should be clear that the cause of the trouble is that we have erroneously generalized results derived for $w > 0$ to the case of $w = 0$. The point is that for $w = 0$, the *leading* piece of the expression inside the square brackets in (2.19) is of order $(l_s/L_s)^2$. For arbitrarily small but finite $(l_s/L_s)^2$ this term should not be discarded. With it one find expressions for $L_{\pm 1}$ that are adequate for removing unwanted states. If $p_- = 0$, for instance, one finds that the state involving γ_{-1} is unphysical, while the one involving β_{-1} is spurious. We thus conclude that only unwound states with transverse polarizations are truly physical — these are the expected newtonian gravitons.

As we have seen in section 2.2, on-shell it is really only states with strictly positive winding that are relevant in the theory, so it is of interest to determine which states are physical for $w > 0$. For this purpose, it is actually quite useful to fall back on more familiar language, rewriting $\gamma, \beta, \tilde{\gamma}, \tilde{\beta}$ in terms of X^+, X^-. Comparing the mode expansions of $L_s\gamma$ and X^+ ($L_s\tilde{\gamma}$ and X^-), it is easy to see that

$$\alpha_n^+ = -i\sqrt{2}n\gamma_n, \qquad \tilde{\alpha}_n^- = -i\sqrt{2}n\tilde{\gamma}_n \qquad \forall \quad n \neq 0\,. \tag{2.28}$$

At the classical level, the effect of the limit (1.1) is to remove the antianalytic (analytic) piece of X^+ (X^-), but from the commutators (2.17) we see that in fact

$$\tilde{\alpha}_n^+ = i\sqrt{2}\tilde{\beta}_n, \qquad \alpha_n^- = i\sqrt{2}\beta_n \qquad \forall \quad n \neq 0\,. \tag{2.29}$$

Similarly, for the zero modes one can deduce that

$$\alpha_0^+ = -\sqrt{2}\left(\frac{wR}{L_s}\right), \qquad \tilde{\alpha}_0^+ = i\sqrt{2}\tilde{\beta}_0 = \sqrt{2}L_s p_-\,, \tag{2.30}$$

$$\tilde{\alpha}_0^- = +\sqrt{2}\left(\frac{wR}{L_s}\right), \qquad \alpha_0^- = i\sqrt{2}\beta_0 = \sqrt{2}L_s p_+\,, \tag{2.31}$$

where we have used (2.22), setting $\lambda = 0$ for simplicity. The Virasoro modes for the entire system then have the standard form,

$$L_n = \frac{1}{2}\sum_m g_{MN} : \alpha_m^M \alpha_{n-m}^N :\,, \tag{2.32}$$

where $M = (+, -, i)$ and $g_{+-} = g_{-+} = 1/2, g_{ii} = 1$. It is convenient to define the usual left- and right-moving momenta $p_{\rm L}^M = (\sqrt{2}/L_s)\alpha_0^M$, $p_{\rm R}^M = (\sqrt{2}/L_s)\tilde{\alpha}_0^M$, i.e.,

$$\begin{aligned} p_{\rm L\it M} &= \left(p_+, -\frac{wR}{L_s^2}, p_i\right), \\ p_{\rm R\it M} &= \left(+\frac{wR}{L_s^2}, p_-, p_i\right). \end{aligned} \tag{2.33}$$

From (2.32) we then have in particular

$$\begin{aligned} L_0 &= \frac{L_s^2}{4} g_{MN} p_{\rm L}^M p_{\rm R}^M + N_{||} + N_\perp\,, \\ \tilde{L}_0 &= \frac{L_s^2}{4} g_{MN} p_{\rm L}^M p_{\rm R}^M + \tilde{N}_{||} + \tilde{N}_\perp\,. \end{aligned} \tag{2.34}$$

We thus see that all expressions have the usual form, except for one peculiarity: if from (2.33) we try to read off left- and right-moving momenta along directions 01 in the 'obvious' way, these would not have the standard form $p^0_{\rm L} = p^0_{\rm R} = p^0$, $p^1_{\rm L,R} = p^1 \pm wR/L_s^2$; in particular, the left and right components of p_0 would not be equal. This is then the main modification that the analysis of [11] brings to light. We should also note that, if one wishes to consider the analog of the above dictionary for the states with zero winding, it is again important to retain terms of order $(l_s/L_s)^2$ in place of the terms involving w in (2.31) and (2.33).

Given the formal agreement with the familiar expressions, the Virasoro constraints can be imposed level by level in the usual manner. In particular, one finds that the polarizations of, e.g., gravitons, are required to be transverse to the momenta as given by (2.33). From this OCQ analysis one concludes that negative-norm states are removed from the physical spectrum in a way consistent with the general no-ghost theorem for Wound string theory, proven in [11] (for $w > 0$) by means of BRST methods. Notice however that, contrary to what the authors of [11] appear to indicate, for $w > 0$ there are physical states in the theory polarized along the 'parallel' directions (i.e., having $N_{||} \neq 0$).

The non-relativistic character of Wound string theory, apparent from the form of the wound string spectrum (2.27) and from the existence of newtonian gravitons, is ultimately expressed by the fact that the action (2.12) is invariant under the galilean group[10] [11]. The invariance under translations and transverse rotations is evident. A galilean boost should have the form

$$X^i \to X^i + \frac{v^i}{2} L_s(\gamma - \tilde{\gamma}), \tag{2.35}$$

with $\gamma, \tilde{\gamma}$ (and therefore X^0, X^1) invariant. It is easy to check that if the remaining fields transform according to

$$\begin{aligned} \beta &\to \beta - \frac{v^i}{L_s}\partial X^i - \frac{\vec{v}^2}{4}\partial(\gamma - \tilde{\gamma})\,, \\ \tilde{\beta} &\to \tilde{\beta} + \frac{v^i}{L_s}\partial X^i + \frac{\vec{v}^2}{4}\partial(\gamma - \tilde{\gamma})\,, \end{aligned} \tag{2.36}$$

then the lagrangian in (2.12) changes only by a total derivative. The effect on the modes can be most easily summarized for all n in terms of $\alpha_n^M, \tilde{\alpha}_n^M$:

$$\begin{aligned} &x^i \to x^i + v^i x^0 \\ &\alpha_n^i \to \alpha_n^i + \frac{v^i}{2}\alpha_n^+\,, && \tilde{\alpha}_n^i \to \tilde{\alpha}_n^i - \frac{v^i}{2}\tilde{\alpha}_n^-\,, \\ &\alpha_n^- \to \alpha_n^- - v^i\alpha_n^i - \frac{\vec{v}^2}{4}\alpha_n^+\,, && \tilde{\alpha}_n^+ \to \tilde{\alpha}_n^+ + v^i\tilde{\alpha}_n^i - \frac{\vec{v}^2}{4}\tilde{\alpha}_n^-\,, \end{aligned} \tag{2.37}$$

[10] This invariance is in accord with the T-duality relation to DLCQ string theory [10, 11].

with α_n^+ and $\tilde{\alpha}_n^-$ invariant. Specializing to $n = 0$, these relations can be seen to imply that the momenta transform according to

$$p_0 \to p_0 - v^i p_i + \frac{\vec{v}^2}{2}\left(\frac{wR}{{L_s}^2}\right), \qquad p_1 \to p_1, \qquad p_i \to p_i - v^i \left(\frac{wR}{{L_s}^2}\right).$$

This is as expected for a galilean boost, with $\mu = wR/{L_s}^2$ playing the role of mass — a fact which was already inferred in [10, 11] from the form of the wound string spectrum (2.27). Notice this explains why the $p_i = 0$ restriction for the unwound states is not incompatible with transverse boosts: these states have vanishing newtonian mass ($\mu = 0$), so their momentum is galilean-invariant.

3. D-branes (mostly transverse)

3.1 Spectrum and collective coordinates

As explained in [10, 11], longitudinal D-branes in Wound string theory comply with the requirement of carrying strictly positive F-string winding by supporting a near-critical electric field on their worldvolume, thus giving rise to a setup which is precisely what is known as NCOS theory [4, 5]. The spectrum of open strings ending on such branes has the standard form (although as explained in section 3.2.1, the relevant open string metric is non-standard). D-branes which are transverse to the compact x^1 direction are also present in the theory [10]; their excitation spectrum is non-standard because of the $w > 0$ requirement. Naively, this restriction appears to indicate that the fluctuation spectrum for transverse branes does not include the usual collective coordinates [10], since the latter are associated with $w = 0$ strings. This conclusion seems rather peculiar, since the objects in question are expected to break translational invariance even after taking the limit. To clarify the situation, let us now explicitly work out the spectrum for open strings ending on a transverse D-brane.

The bosonic worldsheet action in the parent theory (prior to the limit) reads

$$S = \frac{1}{2\pi\alpha'}\int d^2z \left[\eta_{ab}\partial X^a \bar{\partial} X^b + B(\partial X^0 \bar{\partial} X^1 - \partial X^1 \bar{\partial} X^0) + \frac{\alpha'}{L_s^2}\partial X^i \bar{\partial} X^i\right], \tag{3.1}$$

where $a, b = 0, 1$ and $i = 2, \ldots, D-1$. The equations of motion $\partial\bar{\partial}X^\mu = 0$ hold if we enforce boundary conditions at $z = \bar{z}$ such that

$$\begin{aligned}
\delta X^0 \left[(\partial - \bar{\partial})X^0 + B(\partial + \bar{\partial})X^1\right] &= 0\,, \\
\delta X^1 \left[(\partial - \bar{\partial})X^1 + B(\partial + \bar{\partial})X^0\right] &= 0\,, \\
\delta X^i \left[(\partial - \bar{\partial})X^i\right] &= 0\,.
\end{aligned} \tag{3.2}$$

For a *transverse* Dp-brane we expect to be able to choose

$$(\partial - \bar{\partial})X^0 = 0\,, \qquad \delta X^1 = 0\,, \tag{3.3}$$

and Neumann (Dirichlet) boundary conditions along p $(D-p-2)$ of the transverse directions. Notice these conditions are all indeed compatible with (3.2). We can then follow the standard procedure:[11] we write $X^a(z,\bar{z}) = X^a_L(z) + X^a_R(\bar{z})$, and implement the usual doubling trick $X^0_L(\bar{z}) = X^0_R(\bar{z})$, $X^1_L(\bar{z}) = -X^1_R(\bar{z})$ (and similarly for X^i). The mode expansions are

$$
\begin{aligned}
X^0(z,\bar{z}) &= x^0 - i\sqrt{\frac{\alpha'}{2}}\alpha^0_0 \log z\bar{z} + i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\frac{\alpha^0_m}{m}(z^{-m}+\bar{z}^{-m})\,, \\
X^1(z,\bar{z}) &= x^1 - iwR\log\frac{z}{\bar{z}} + i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\frac{\alpha^1_m}{m}(z^{-m}-\bar{z}^{-m})\,,
\end{aligned} \tag{3.4}
$$

and the expressions for the conjugate momenta,

$$
\begin{aligned}
\Pi_0 \equiv i\frac{\partial\mathcal{L}}{\partial\dot{X}^0} &= \frac{i}{2\pi\alpha'}\left[z\partial X_0 + \bar{z}\bar{\partial}X_0 - Bz\partial X_1 + B\bar{z}\bar{\partial}X_1\right], \\
\Pi_1 \equiv i\frac{\partial\mathcal{L}}{\partial\dot{X}^1} &= \frac{i}{2\pi\alpha'}\left[z\partial X_1 + \bar{z}\bar{\partial}X_1 - Bz\partial X_0 + B\bar{z}\bar{\partial}X_0\right],
\end{aligned} \tag{3.5}
$$

imply that the corresponding zero modes are related through

$$
p_0 = -\frac{1}{\sqrt{2\alpha'}}\alpha^0_0 - \frac{wRB}{\alpha'}\,, \qquad p_1 = 0\,. \tag{3.6}
$$

We should note that the *total* longitudinal momentum does not vanish, due to a net contribution from the non-zero modes:

$$
P_1 \equiv \int_0^\pi d\sigma^1\,\Pi_1 = -\frac{i}{\pi\sqrt{2\alpha'}}\sum_{m\neq 0}\frac{e^{-m\sigma^2}}{m}(\alpha^1_m + B\alpha^0_m)\,. \tag{3.7}
$$

(The center of mass coordinate $\bar{x}^1 \equiv (1/\pi)\int d\sigma^1\, X^1$ receives a similar time-dependent contribution from the non-zero modes.) This is just a reflection of the fact that the brane breaks translational invariance along x^1, so the total longitudinal momentum is not conserved (this is true even if $B=0$). Notice on the other hand that the time average of P_1 (with respect to $\tau = -i\sigma^2$) vanishes — just as it should, since the string cannot wander away from the brane. The time-dependence of P_1 should not be cause for distraction: it is p_1, the zero mode of Π_1, that is canonically conjugate to x^1.

It follows from the above that

$$
L^{||}_0 = \frac{1}{2}\sum_m \eta_{ab} : \alpha^a_{-m}\alpha^b_m := -\alpha'(p_0)^2 - 2p_0 wRB + \frac{(wR)^2}{\alpha'}(1-B^2) + N_{||}\,, \tag{3.8}
$$

[11] For longitudinal D-branes things are actually not this simple — see, e.g., [31]–[34]. As is well-known by now, the end result in that case can be succinctly summarized by introducing an effective open string metric, coupling constant, and non-commutativity parameter [3].

which together with

$$L_0^{\perp} = L_s^2 p_{\perp}^2 + N_{\perp} \tag{3.9}$$

implies that the spectrum is identical to that of closed strings (with $p_1 = 0$),

$$p_0 = -\frac{BwR}{\alpha'} + \sqrt{\left(\frac{wR}{\alpha'}\right)^2 + \frac{L_s^2 p_{\perp}^2}{\alpha'} + \frac{N_{||} + N_{\perp}}{\alpha'}}\,. \tag{3.10}$$

The effect of the limit (1.1) + (2.10) on this spectrum was determined in [9, 10]: states with $w > 0$ will have a finite energy

$$p_0 = \lambda \frac{wR}{L_s^2} + \frac{L_s^2 p_{\perp}^2}{2wR} + \frac{N_{||} + N_{\perp}}{2wR}\,, \tag{3.11}$$

whereas states with $w = 0$ will have a finite energy only if $p_{\perp} = N_{||} + N_{\perp} = 0$, in which case $p_0 = 0$.

The positively wound states were anticipated in [10]; they are the mechanism through which the transverse brane can comply with the requirement of carrying positive F-string winding. The unwound strings are an exception to this requirement, just like the newtonian gravitons discussed in section 2. The exception is important here since it resolves the puzzle discussed at the beginning of this section: the $w = 0$ strings are present only at oscillator level $N = 0$, so these are precisely the modes giving rise to the standard D-brane gauge field and collective coordinates. Notice that the constraint $p_{\perp} = 0$ applies to all transverse directions, including those along the Dp-brane. For a D-particle this constraint is automatically satisfied. For $p \geq 1$, it implies that the 'photons' on the brane, just like the gravitons in the bulk, are newtonian: they propagate at infinite speed, and mediate instantaneous interactions between the charges associated with string endpoints. Like the gravitons, they are negligible as asymptotic states.

It is also interesting to consider strings connecting two parallel transverse D-branes. If the branes are separated by a distance L along the longitudinal direction, the X^0, X^1 mode expansions for a string extending from one brane to the other are as in (3.4), with wR replaced by $L/2\pi$. These strings can thus be regarded as a particular case of the wound open strings considered above, with fractional winding number $w \equiv L/(2\pi R)$. In particular, their energy spectrum in the limit (1.1) + (2.10) is given by (3.11). So in contrast with the rest of the objects in Wound string theory, whose newtonian masses are integer multiples of R/L_s^2, the mass of these strings is an arbitrary (positive) real number, $\mu = L/(2\pi L_s^2)$, which can remain finite in the decompactification limit $R \to \infty$. Notice that $\mu \to 0$ when the branes approach one another, so again the energy of these strings remains finite only if $p_{\perp} = N = 0$.

Before closing this section, let us address the question of whether the open string sector analyzed above, describing the excitations of a transverse D-brane, can interact consistently with the closed string sector of Wound string theory. For the case of a longitudinal D-brane, (i.e., NCOS theory,) consistency was proven in [9] by showing that a non-planar one-loop amplitude describing the scattering of open strings attached to the brane has poles at energies which correspond precisely to the closed string spectrum (2.27). For the case of a transverse D-brane it turns out to be easier to establish consistency — the argument goes as follows. Consider the non-planar annulus diagram in the parent string theory (before taking the limit). Notice first that the boundary conditions (3.3) do not involve the B-field, so the propagator on the annulus is the standard one [35]. This implies that the annulus amplitude (see e.g. [36, eqs. (3.1)–(3.4)]) depends on B only through its effect on the masses of the open strings in the loop, which are summed over in the usual partition function. From the analysis in the present subsection, we know that this effect is extremely simple: as seen in (3.10), the presence of the B field merely effects an energy subtraction proportional to the winding. For any finite value of δ, the amplitude in question will clearly have poles at the energies given by the closed string spectrum (2.2), where B enters through the same energy subtraction. Taking the limit (1.1) + (2.10), then, we obtain an amplitude whose poles necessarily coincide with the closed string spectrum (2.27). In short, the point is that in this aspect, as in all others, the consistency of Wound string theory is guaranteed by the consistency of its parent theory.

3.2 Worldsheet perspective

It is interesting to ask how transverse D-branes are seen in the Gomis-Ooguri formalism [11]. To answer this question, one must consider the worldsheet action (2.12) in the presence of a boundary. The equations of motion are then seen to imply the boundary conditions [11]

$$\delta\gamma\left(\beta-\frac{\lambda}{2}\bar{\partial}\tilde{\gamma}\right)+\delta\tilde{\gamma}\left(-\tilde{\beta}+\frac{\lambda}{2}\partial\gamma\right)=0\,. \tag{3.12}$$

We will consider the theory formulated on the upper-half plane; the boundary is then at $z=\bar{z}$. There should be at least two ways of satisfying the above condition, corresponding to longitudinal and transverse D-branes. The former type of D-brane was shown in [11] to lead to the standard NCOS setup [4, 5]. Let us first review that case, to clarify the dependence of the open string spectrum on the parameters of the theory.

3.2.1 Longitudinal D-branes

For D-branes extended along the longitudinal direction it is clear that neither γ nor $\tilde{\gamma}$ should satisfy Dirichlet boundary conditions, so we must demand that [11]

$$\beta|_{z=\bar{z}}=\frac{\lambda}{2}\bar{\partial}\tilde{\gamma}\Big|_{z=\bar{z}}\,,\qquad \tilde{\beta}|_{z=\bar{z}}=\frac{\lambda}{2}\partial\gamma\Big|_{z=\bar{z}}\,. \tag{3.13}$$

Before proceeding further it is important to realize that for this type of D-brane it is not enough to specify the value of the background B_{01}-field. A longitudinal D-brane placed in such a field describes a bound state of a D-brane and a definite number N of fundamental strings. Since the total F-string winding number W is conserved by interactions, one can consistently restrict attention to a sector of the theory with a particular value of W, but included in this sector are states where the D-brane carries different winding numbers [10]. In other words, each D-brane can carry a different electric field $E \equiv 2\pi\alpha' F_{01}$ on its worldvolume, whose value must be specified.

The combination $E+B$ determines the number N of F-strings bound to the Dp-brane through the flux quantization condition (see, e.g., [5])

$$N g_s = \left(\frac{r_2 \cdots r_p}{l_s^{p-1}}\right) \sqrt{-g_{(p+1)}} \frac{|g^{00}| g^{11}(E+B)}{\sqrt{1-|g^{00}|g^{11}(E+B)^2}}\,, \tag{3.14}$$

where $g_{(p+1)}$ is the determinant of the induced metric on the brane. In the limit (1.1), this condition can be seen to imply

$$E+B = 1 - \frac{1}{2\nu^2 G_s^2}\left(\frac{l_s}{L_s}\right)^2, \qquad \nu \equiv \frac{N L_s^{p-1}}{r_2 \cdots r_p}\,. \tag{3.15}$$

The description is physically most transparent in the gauge $B=0$; it is then evident that the value of E (or equivalently, the number density ν) should be specified separately for each brane [10]. In this description, the infinite contribution of F-string winding to the energy of each object of the theory must be subtracted by hand. Alternatively, as explained in [10], the subtraction can be implemented by gauging E away from the brane and into the bulk, in the form of a B-field. The point we are emphasizing here is that with this single operation it is not possible to set $E=0$ on all possible longitudinal D-branes in the theory. After the gauge transformation B is given by (2.10), so the electric field that remains on the D-brane worldvolume is $E = [\lambda - 1/(2\nu^2 G_s^2)](l_s/L_s)^2$. This (dimensionless) field vanishes in the limit, but it leaves behind a finite boundary action

$$\int d\tau\, A_a \partial_\tau X^a = \frac{1}{2\pi\alpha'}\int d\tau\, E X^0 \partial_\tau X^1 = \frac{1}{2\pi L_s^2}\int d\tau\, \left[\lambda - \frac{1}{2\nu^2 G_s^2}\right] X^0 \partial_\tau X^1\,. \tag{3.16}$$

When this is added to the bulk worldsheet action (2.12) the boundary conditions are modified: instead of (3.13) one must enforce

$$\beta|_{z=\bar z} = \frac{1}{4\nu^2 G_s^2}\bar\partial\tilde\gamma\Big|_{z=\bar z}\,, \qquad \tilde\beta|_{z=\bar z} = \frac{1}{4\nu^2 G_s^2}\partial\gamma\Big|_{z=\bar z}\,. \tag{3.17}$$

Having understood that the spectrum of open strings attached to a longitudinal D-brane is controlled not by the free parameter λ appearing in (2.10), but by the definite quantity $1/(2\nu^2 G_s^2)$, where ν is the F-string number density defined in (3.15), we can proceed with the review of [11]. As explained there, in view of (3.17) it

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is natural to implement a 'doubling trick', extending $\beta(z)$ and $\tilde{\beta}(\bar{z})$ to the entire complex plane by setting, for all z in the upper-half plane,

$$\beta(\bar{z}) = \frac{1}{4\nu^2 G_s^2}\bar{\partial}\tilde{\gamma}(\bar{z})\,, \qquad \tilde{\beta}(z) = \frac{1}{4\nu^2 G_s^2}\partial\gamma(z)\,. \tag{3.18}$$

The boundary conditions (3.17) then amount to requiring continuity of $\beta, \tilde{\beta}$ across the real axis. We thus have the mode expansions

$$\begin{aligned}
\beta(z) = \sum_{n=-\infty}^{\infty} \beta_n z^{-n-1} &\Longrightarrow \tilde{\gamma}(\bar{z}) = 4\nu^2 G_s^2 \beta_0 \log \bar{z} + \gamma_0 - 4\nu^2 G_s^2 \sum_{n\neq 0} \frac{\beta_n}{n} \bar{z}^{-n}\,, \\
\tilde{\beta}(\bar{z}) = \sum_{n=-\infty}^{\infty} \tilde{\beta}_n \bar{z}^{-n-1} &\Longrightarrow \gamma(z) = 4\nu^2 G_s^2 \tilde{\beta}_0 \log z + \gamma_0 - 4\nu^2 G_s^2 \sum_{n\neq 0} \frac{\tilde{\beta}_n}{n} z^{-n}\,,
\end{aligned} \tag{3.19}$$

the energy-momentum tensor

$$T^{||}(z) = 4\nu^2 G_s^2 :\beta(z)\tilde{\beta}(z):\,,$$

and the Virasoro modes

$$L_n^{||} = 4\nu^2 G_s^2 \sum_l :\beta_l \tilde{\beta}_{n-l}:\,. \tag{3.20}$$

The non-zero commutators are

$$[\gamma_0, \beta_0] = 1, \qquad [\tilde{\gamma}_0, \tilde{\beta}_0] = 1, \qquad [\tilde{\beta}_n, \beta_m] = \frac{1}{4\nu^2 G_s^2} n\delta_{n+m}\,. \tag{3.21}$$

Using (2.21), we deduce that

$$p_+ \equiv \frac{1}{2}(p_0 + p_1) = \frac{i\beta_0}{L_s}, \qquad p_- \equiv \frac{1}{2}(-p_0 + p_1) = \frac{i\tilde{\beta}_0}{L_s} \tag{3.22}$$

Comparing the expansions for $\gamma = X^+/L_s$ and $\tilde{\gamma} = X^-/L_s$ in (3.19) with the standard open string mode expansion we see that $L_s\gamma_0 = x^+$, $L_s\tilde{\gamma}_0 = x^-$, and it is also natural to define

$$\alpha_m^- = i2\sqrt{2}\nu^2 G_s^2 \beta_m\,, \qquad \alpha_m^+ = i2\sqrt{2}\nu^2 G_s^2 \tilde{\beta}_m \qquad \forall \quad m \neq 0\,. \tag{3.23}$$

We can then rewrite the commutators (3.21) in the form

$$\begin{aligned}
[x^+, p_+] &= i = [x^-, p_-]\,, \\
[\alpha_n^+, \alpha_{-n}^-] &= 2\nu^2 G_s^2 n = [\alpha_n^-, \alpha_{-n}^+] \qquad \forall \quad n > 0\,.
\end{aligned} \tag{3.24}$$

Together with the contribution from the transverse part of the system, we thus clearly have the standard open string commutators,

$$[x^M, p^N] = iG^{MN}, \qquad [\alpha_m^M, \alpha_n^N] = G^{MN} m\delta_{m+n}, \tag{3.25}$$

where $M, N = +, -, 2, \ldots, D-1$, and we have introduced the Seiberg-Witten [3] open string metric $G_{+-} = 1/(2\nu^2 G_s^2)$, $G_{ij} = \delta_{ij}$.

From (3.20) we have

$$L_0^{||} = 4\nu^2 G_s^2 L_s^2 p_+ p_- + \frac{1}{2\nu^2 G_s^2} \sum_{n=1}^{\infty} \alpha_{-n}^+ \alpha_n^- + \frac{1}{2\nu^2 G_s^2} \sum_{n=1}^{\infty} \alpha_{-n}^+ \alpha_n^- \,, \tag{3.26}$$

which means the open string spectrum takes the usual form,

$$L_s^2 G^{MN} p_M p_N + \sum_{n=1}^{\infty} \alpha_{-n}^M \alpha_n^N G_{MN} = 0 \,, \tag{3.27}$$

except that the metric is non-standard. This mass-shell condition can be written out explicitly as

$$\nu^2 G_s^2 \left[(p_0)^2 - (p_1)^2 \right] - p_\perp^2 = \frac{N_{\rm osc}}{L_s^2} \,, \tag{3.28}$$

where $N_{\rm osc}$ denotes the number operator for the oscillators. The standard NCOS convention [4, 5] is to perform a ν-dependent rescaling of the transverse closed string metric such that the factor $\nu^2 G_s^2$ is common to all terms in the left-hand side, and then absorb that factor through a redefinition of the effective string length, to be left with

$$(p_0)^2 - (p_1)^2 - k_\perp^2 = \frac{N_{\rm osc}}{\alpha_e'} \,, \qquad \alpha_e' \equiv \nu^2 G_s^2 L_s^2 \,. \tag{3.29}$$

The point emphasized in [10] and reiterated here is that, while these steps are sensible from the NCOS perspective, where one restricts attention to a particular Dp-brane setup (with a specific value for p and ν), they are not convenient when dealing with the complete Wound string theory, where one should employ the same closed string metric and string length for all possible configurations. Doing this one arrives at expressions like (3.28), which manifestly displays the dependence of the open string spectrum on the relevant parameters.

Notice that, as advertised in section 2.3, the open string spectrum (3.28) is independent of the parameter λ appearing in (2.10), so one is certainly free to set λ to any desired value. The simplest choice is $\lambda = 0$, which amounts to removing the first term from the energy spectra (2.27) and (3.11).

3.2.2 Transverse D-branes

Besides (3.17), an 'obvious' way to satisfy (3.12) would be to demand that $\delta\gamma = 0$ (and either $\delta\tilde{\gamma} = 0$ or $\tilde{\beta} = 0$) at $z = \bar{z}$. There is a problem with these boundary conditions, however: since $\gamma(z)$ is analytic, requiring it to be constant on the boundary forces it to be constant everywhere. Physically, this seems counterintuitive, because it would mean that the entire body of the string (and not just the endpoints) has a fixed position along x^+. Mathematically, the problem is that these boundary conditions would be incompatible with the algebra (2.17), because they require setting the *creation* operators $\gamma_{n<0}$ to zero. The way out is to require $\gamma+\tilde{\gamma}$ (and not γ or $\tilde{\gamma}$ separately) to be constant on the real axis — this is precisely as expected for a D-brane orthogonal to the x^1 direction. We then have $\delta\gamma=-\delta\tilde{\gamma}$, or in other words $\partial\gamma=-\bar{\partial}\tilde{\gamma}$, at $z=\bar{z}$. To satisfy (3.12) we must then set $\beta=-\tilde{\beta}$ at the boundary. Notice this implies that $T^{||}(z)$ and $\tilde{T}^{||}(\bar{z})$ agree there (see (2.18)), as needed for consistency. The mode expansions are

$$\begin{aligned}\gamma(z) &= -i\left(\frac{2wR}{L_s}\right)\log z+\sum_n \gamma_n z^{-n}\,, \qquad \beta(z)=+\sum_n \beta_n z^{-n-1}\,,\\ \tilde{\gamma}(\bar{z}) &= +i\left(\frac{2wR}{L_s}\right)\log \bar{z}-\sum_n \gamma_n \bar{z}^{-n}\,, \qquad \tilde{\beta}(\bar{z})=-\sum_n \beta_n \bar{z}^{-n-1}\,.\end{aligned} \tag{3.30}$$

Using this in (2.21), we can extract the zero modes

$$p_0=i\frac{\beta_0}{L_s}-\lambda\left(\frac{wR}{L_s^2}\right), \qquad p_1=0\,. \tag{3.31}$$

From (2.18) and (3.30) it follows that

$$L_0^{||}=-i\beta_0\left(\frac{2wR}{L_s}\right)+N_{||}\,. \tag{3.32}$$

Together with (3.31) and (3.9), this implies that the condition $L_0=0$ is equivalent to

$$p_0=\lambda\frac{wR}{L_s^2}+\frac{L_s^2 p_\perp^2}{2wR}+\frac{N_{||}+N_\perp}{2wR}\,, \tag{3.33}$$

which is in agreement with (3.11). The strings with 'fractional winding' (extending between two different D-branes) analyzed in section 3.1 can be accommodated here as well, simply by taking w to be an arbitrary (positive) real number. The treatment of the exceptional $w=0$ states is parallel to the closed string case analyzed in section 2.3.

4. Supergravity description

We will now discuss some of the results of [10, 11] and the previous sections from the point of view of supergravity duals. For this purpose we should consider a system of $K\gg 1$ coincident longitudinal Dp-branes in Wound string theory. As explained in [10, 6], this is precisely the setup known as $p+1$ NCOS theory [4, 5]. The supergravity duals for NCOS theories were worked out in [5, 19, 37]; from the point of view of the parent string theory they are 'near-horizon' limits of D-branes with a worldvolume electric field. We emphasize here that in this case the duality is between a supergravity background and a theory of strings (the NCOS/Wound theory), instead of a field theory as in the usual case [38].

In the presence of the D-branes, and for a finite radius R of the longitudinal direction, Wound string theory contains not only open strings attached to the branes, but also positively-wound closed strings which live in flat space [9, 10, 11]. In the decompactification limit $R \to \infty$, the worldvolume theory on the branes (still a full-fledged string theory) decouples from the closed strings.

Now, it is important to realize that the 'near-horizon' limit that defines the supergravity duals of [5, 19] is different from the familiar AdS/CFT scaling [38]. In particular, the NCOS limit (1.1) keeps transverse *proper* distances fixed in units of l_s, whereas the Maldacena scaling requires that $r \ll l_s$. It is therefore interesting to ask exactly what objects are kept by the scaling in the supergravity description.

To address this question, consider the supergravity background dual to $p+1$ NCOS [19],

$$\begin{aligned}
\frac{ds^2}{l_s^2} &= \frac{1}{L_s^2} H^{1/2} \frac{K^2}{G_s^2 \nu^2} \frac{u^{7-p}}{R_D^{7-p}} \left(-dX_0^2 + dX_1^2\right) + \frac{1}{L_s^2} H^{-1/2} \left(dX_2^2 + \cdots + dX_p^2\right) + \\
&\quad + \frac{1}{L_s^2} H^{1/2} \left(dX_{p+1}^2 + \cdots + dX_{d-1}^2\right) \\
g_{\text{eff}} &= g_s e^{\phi} = H^{\frac{5-p}{4}} \frac{K}{\nu} \frac{u^{\frac{7-p}{2}}}{R_D^{\frac{7-p}{2}}} \\
\frac{B_{01}}{l_s^2} &= \frac{1}{G_s^2 L_s^2} \frac{K^2}{\nu^2} \frac{u^{7-p}}{R_D^{7-p}} ,
\end{aligned} \tag{4.1}$$

where

$$\begin{aligned}
H &= 1 + \frac{R_D^{7-p}}{u^{7-p}} \\
u^2 &= X_{p+1}^2 + \cdots + X_{d-1}^2 \\
R_D^{7-p} &= \frac{1}{7-p} \frac{K^2}{\nu} \frac{L_s^{7-p}}{V(S^{8-p})} ,
\end{aligned} \tag{4.2}$$

and $V(S^{8-p})$ is the volume of the S^{8-p} sphere. Notice that we have rewritten the expressions of [19] in the units and notation of the previous sections. In particular, X_1 is regarded as being compactified on a circle of radius R, and ν is the density of fundamental strings bound to the K Dp-branes, as defined in (3.15).

It is easy to see that a signal propagating outwards in this background at the speed of light, that is, obeying

$$\frac{du}{dX_0} = \frac{K}{G_s \nu} \frac{u^{\frac{7-p}{2}}}{R_D^{\frac{7-p}{2}}} , \tag{4.3}$$

reaches the boundary $u = \infty$ in a finite time (for $p < 5$), whereas a massive particle will never be able to reach $u = \infty$. In this sense the background (4.1) behaves like a box, just like AdS space.

On the other hand, due to the presence of the B-field, some of the properties of the background (4.1) are certainly very different from those of AdS. This difference can be seen most explicitly by considering a fundamental string wound around the X^1 direction. Besides the usual Nambu-Goto term, the relevant worldsheet action includes of course a coupling to the B-field, and so reads

$$\begin{aligned} S &= \frac{1}{2\pi l_s^2}\int d\tau d\sigma \left(\sqrt{h} - B_{01}\partial_\sigma X_1\right) \\ &= \int dX_0 \, \frac{K^2}{\nu^2}\frac{R}{G_s^2 L_s^2}\frac{u^{7-p}}{R_D^{7-p}}(H^{1/2}-1)\,. \end{aligned} \tag{4.4}$$

In the second step we have assumed a time-independent configuration, and have made the static gauge choice $X_0 = \tau$, $X_1 = \sigma R$. From here we can infer that the string lives in a potential

$$V(u) = \frac{K^2}{\nu^2}\frac{1}{G_s^2 L_s^2}R\frac{u^{7-p}}{R_D^{7-p}}\left(\sqrt{1+\frac{R_D^{7-p}}{u^{7-p}}}-1\right). \tag{4.5}$$

For $u \to 0$ we have $V(u) \to 0$, whereas near the boundary

$$V(u) \simeq \frac{K^2}{\nu^2}\frac{R}{G_s^2 L_s^2}\left(\frac{1}{2} - \frac{1}{8}\frac{R_D^{7-p}}{u^{7-p}}\cdots\right) \tag{4.6}$$

The second term gives an attractive newtonian potential, whereas the first gives the energy necessary to pull the string out to infinity, which is then

$$\Delta E = \frac{1}{2}\frac{K^2}{\nu^2}\frac{R}{G_s^2 L_s^2}\,. \tag{4.7}$$

The fact that the string needs only a finite energy to reach the boundary is due to a cancellation between the two terms in (4.4), and so depends crucially on the relative sign between them, which reflects a particular choice of orientation. The sign chosen in (4.4) is appropriate for a string which winds in the direction of the B-field. The potential for an oppositely wound string would diverge at infinity. This restriction to $w > 0$ suggests that the string in question is the supergravity counterpart of a closed string in the Wound/NCOS theory. Strings of this type are known as long strings [39, 40, 41], and have appeared before in other discussions of supergravity duals.

We can verify this picture quantitatively by noting that in the Wound/NCOS side of the duality, the expression (4.7) should give the energy necessary to pull one closed string out of the F1/Dp bound state. The tension of a bound state of K Dp-branes and ν fundamental strings per unit $p-1$ volume in the limit (1.1) is [9, 10]

$$\hat{T}_{K,\nu} = \frac{K^2}{2(2\pi)^p \nu G_s^2 L_s^{p+1}}\,, \tag{4.8}$$

so the total energy of the bound state is

$$E_{K,\nu} = \frac{1}{2}\frac{K^2}{\nu}\frac{R}{(G_sL_s)^2}\frac{r_2\cdots r_p}{L_s^{p-1}}\,, \tag{4.9}$$

where $r_2,\ldots,r_p$ denote the transverse radii. When the number N of fundamental strings in the bound state changes by one, the number density ν can be seen from (3.15) to change by $\delta\nu = L_s^{p-1}/(r_2\cdots r_p)$, resulting in an energy change which is exactly the same as in (4.7). We thus confirm that the long strings living near the boundary of the supergravity background correspond to the wound closed strings of the dual picture. The usual short strings are essentially confined to the region $u < R_D$; they correspond to the open strings living on the brane, as in the familiar AdS/CFT mapping.[12] That the open/closed strings in NCOS theories have properties analogous to those of the AdS_3 short/long strings discussed in [41] had been noted already by Klebanov and Maldacena [9].

Further evidence for the identification of long strings with closed strings is given by the fact that for $u \gg R_D$ (i.e., close to the boundary), the worldsheet action (4.4) for a long string reduces to the Gomis-Ooguri [11] action for a Wound string, Eq. (2.12). To prove this assertion, we can first note that in the region $u \gg R_D$, the background (4.1) reduces to

$$\begin{aligned}
\frac{ds^2}{l_s^2} &= \frac{1}{L_s^2}\left[\frac{K^2}{G_s^2\nu^2}\frac{u^{7-p}}{R_D^{7-p}}\left(-dX_0^2+dX_1^2\right)+dX_2^2+\cdots+dX_{d-1}^2\right],\\
g_{\mathrm{eff}} &= g_s e^{\phi} = \frac{K}{\nu}\frac{u^{\frac{7-p}{2}}}{R_D^{\frac{7-p}{2}}}\,,\\
\frac{B_{01}}{l_s^2} &= \frac{1}{G_s^2L_s^2}\frac{K^2}{\nu^2}\frac{u^{7-p}}{R_D^{7-p}}\,.
\end{aligned} \tag{4.10}$$

In Polyakov form, the action (4.4) for a long string in this background reads

$$\begin{aligned}
S = \frac{1}{L_s^2}\int d\tau\, d\sigma\Bigg[&\frac{K^2}{G_s^2\nu^2}\frac{u^{7-p}}{R_D^{7-p}}\left(-\eta^{ab}\partial_aX_0\partial_bX_0+\eta^{ab}\partial_aX_1\partial_bX_1+2\epsilon^{ab}\partial_aX_0\partial_bX_1\right)+\\
&+\eta^{ab}\left(\partial_aX_2\partial_bX_2+\ldots+\partial_aX_{d-1}\partial_bX_{d-1}\right)\Bigg].
\end{aligned} \tag{4.11}$$

In the region $u \gg R_D$ we can define a small parameter

$$\delta = \frac{G_s^2\nu^2}{K^2}\frac{R_D^{7-p}}{u^{7-p}}\,. \tag{4.12}$$

Upon making the identifications $l_s^2 = L_s^2\delta$ for the string length, and

$$g_{\mathrm{eff}} = \frac{K}{\nu}\frac{u^{\frac{7-p}{2}}}{R_D^{\frac{7-p}{2}}} = G_s\frac{1}{\sqrt{\delta}} \tag{4.13}$$

[12] To be precise, in the AdS/CFT correspondence only the massless modes of the open strings remain in the limit, whereas here the whole tower of modes is kept.

for the string coupling, it becomes clear that the limit $u \to \infty$ is the same as (1.1). Since this is the limit that led to (2.12), one can evidently use the Lagrange-multiplier trick of Gomis and Ooguri, to obtain the β-γ system as in [11]. In a sense, this result is not surprising, since the limit (1.1) plays a role in the derivation of the supergravity background (4.1). The point which is worth emphasizing is that, whereas short strings essentially see this background as a box, the above result shows that long strings see an asymptotically flat space.

The above analysis makes it rather clear that even though superficially (from the point of view of the parent string theory) the limit (1.1) used to obtain the supergravity background (4.1) appears to be a near-horizon limit, it is actually quite different in nature from the Maldacena limit [38]. The calculations of [5, 19] are best regarded as a derivation of the macroscopic field configuration produced by a large number of longitudinal D-branes in Wound string theory. From the point of view of the Wound theory, the duality in question *does not involve any additional limits*, so it is simply based on the possibility of describing the D-branes of this theory either by means of open strings or through the supergravity background they create. The analogous duality in a conventional string theory would be not the AdS/CFT correspondence, but the equivalence between the full asymptotically flat black brane backgrounds of [42] and Polchinski's open-string description of D-branes [43]. This equivalence is the starting point of Maldacena's analysis [38], and of the various recent attempts to obtain a holographic dual for the full asymptotically flat backgrounds [44, 45, 46, 47, 48].

Given the results of section 2, we would expect the fluctuations of the background (4.1) to include a special class of unwound strings, corresponding to the newtonian gravitons of Wound string theory. It is easy to see how they appear. Consider the metric

$$\frac{ds^2}{l_s^2} = \eta_{\mu\nu} dx^\mu dx^\nu \,, \tag{4.14}$$

where

$$\eta_{+-} = \frac{1}{l_s^2} \,, \qquad \eta_{ij} = \delta_{ij} \,. \tag{4.15}$$

For a perturbation $h_{\mu\nu}$ around this background, the Einstein equation reduces to

$$R_{\mu\nu} = \frac{1}{2}\eta^{\delta\sigma} \left(h_{\nu\sigma,\mu\delta} + h_{\mu\sigma,\nu\delta} - h_{\mu\nu,\delta\sigma} - h_{\delta\sigma,\mu\nu} \right) = 0 \,, \tag{4.16}$$

from where it is clear that terms containing $\eta^{+-} \sim l_s^2$ disappear in the limit $l_s^2 \to 0$, allowing for more solutions. Some of these solutions will not be physical, in the sense that they will not correspond to any solution with $l_s^2 \neq 0$. As we emphasized in section 2.3, for the unwound states l_s^2 should be considered to be small but non-vanishing, so solutions of this type should be discarded. Solutions describing coordinate transformations are of course associated with null states. The remaining solutions can be seen to correspond to the newtonian gravitons discussed in section 2.

It is also interesting to ask how the results of the present section generalize to the transverse D-branes of Wound string theory. One can determine the supergravity background that these branes generate by starting with the standard D-brane solution in the parent string theory [42], (with a superposed B_{01}-field,) and then taking the limit (1.1) + (2.10). As usual, to comply with the requirement of periodicity along the compact longitudinal dimension, one can either set up a periodic array of localized sources in the covering space, or consider a longitudinally smeared source. Either way, one obtains a metric whose 00 and 11 components diverge everywhere relative to the others, in contrast with the longitudinal D-brane case (4.1), where they only diverge asymptotically. This divergence can be dealt with using the methods of [11], and it indicates that, just like in the flat case (1.1), only positively wound strings can propagate in this background. We will elaborate on this set of issues elsewhere.

5. Conclusions

We have learned that, in a very specific sense, gravity is present in Wound string theory — as discussed in section 2.1, unwound strings with zero oscillator number, including a graviton, are part of the theory. On-shell, these strings are forced to have vanishing transverse momentum, which as explained in section 2.2 means that they are irrelevant as asymptotic states. Off-shell, their transverse momentum is arbitrary, and as the only massless states in the theory, they are the mediators of all long-range interactions. As a result of the scaling of the metric in the limit (1.1) that defines the theory, these messenger particles propagate at infinite speed, and so the interactions they mediate are instantaneous [11]. This is an expression of the non-relativistic nature of Wound string theory, apparent also in the T-dual DLCQ description.

The presence of gravity in Wound string theory should not really come as a surprise — this is, after all, the reason why it has been possible to discuss supergravity duals for the longitudinal D-branes of this theory[13] [5, 19]. As one would expect, a collection of a large number of objects in the theory (wound strings, D-branes or NS5-branes) does set up macroscopic gravitational and Kalb-Ramond fields (and possibly others). The point made clear by the results of [11] and the present paper is that these fields are newtonian in character: they follow the source instantaneously. So, even though gravity is present in the theory, it is still true that there are no finite-time fluctuations of the gravitational field — there are no gravitons, in the traditional sense of the word. Such spacetime fluctuations are at the root of the traditional conceptual difficulties in attempts to understand quantum gravity, so the

[13]In this connection we would like to emphasize the point, made already in section 4, that from the perspective of the Wound theory the duality in question is analogous not to the AdS/CFT correspondence, but to Polchinski's identification of D-branes with R-R black branes.

hope remains that Wound string/NCOS theory, devoid of such complicating features, could facilitate our understanding of the underlying structure of string theory.

Note that the above statements about gravity in Wound string theory can be generalized to all of the other Wrapped brane theories; e.g., Wrapped M2-brane theory [10, 11] contains newtonian gravitons which, on-shell, can carry momentum only along the two 'longitudinal' directions (i.e., the directions along which the metric is not scaled to zero).

In section 3.1 we worked out the excitation spectrum for transverse D-branes, and found that open strings with $w = 0$ give rise to the expected gauge field and collective coordinates. The non-relativistic character of the theory is apparent here from the fact that there are no waves on these branes: the photons on their worldvolume, just like the gravitons in the bulk, are newtonian, and the $p_\perp = 0$ restriction on the scalars implies that the branes can only be translated rigidly. As in the case of gravity, a macroscopic source can set up a non-trivial gauge/scalar field configuration, which follows the source instantaneously. It would be interesting, then, to look for the analog of Born-Infeld strings [49, 50] in this context.

Besides explaining how all of the above conclusions follow from a direct examination of the limit (1.1) that defines the theory, we have shown how they can be extracted from the worldsheet formalism developed by Gomis and Ooguri [11]. As explained in section 2.3, a direct application of the lagrangian of [11] to the $w = 0$ states would lead to erroneous conclusions; special care must be taken to retain terms which would be subleading for $w > 0$, but are necessary to project out negative-norm states in the case of unwound strings. We have also verified in section 3.2.2 that the Gomis-Ooguri formalism yields the correct spectrum for open strings ending on transverse D-branes.

Some additional remarks regarding the approach of [11] have been made at various places. In section 2.3 it was pointed out that the Gomis-Ooguri action (2.12) attains its simplest form if from the beginning we fine-tune the B-field to its critical value (i.e., set the free parameter λ in (2.10) equal to zero). It was also shown there that the treatment of [11] can easily be extended to the worldsheet fermions, resulting in the action (2.14), where ψ^0, ψ^1 have been traded for a system of anticommuting ghosts. Setting $B = 1$ would appear to cause difficulties when longitudinal D-branes are present, so in section 3.2.1 we have retraced the steps of [11] for that case to explain why there is in fact no problem. The point is that, in addition to B, for each longitudinal brane one must specify the value of the worldvolume electric field. When this is done, the open string spectrum is understood to be independent of the free parameter λ; its explicit dependence on the parameters G_s, L_s of the Wound theory and the F-string density ν on the brane can be seen in (3.28). In this connection we stress the point, made already in [10], that the NCOS conventions are not convenient when dealing with the full Wound theory, because they obscure the dependence of physical quantities on the relevant parameters.

Finally, in section 4 we considered the known supergravity duals for Wound string theory in the presence of longitudinal D-branes (i.e., NCOS theory) [5, 19]. We observed in particular that the supergravity description accounts not only for the open strings attached to the branes, but also for the wound strings which can move away from them. Whereas the former are described by the usual local perturbations of the background (short strings), the latter are visible as long strings analogous to those of [39, 40, 41]. That the open/closed strings in NCOS theories have properties analogous to those of the AdS_3 short/long strings discussed in [41] had been noted already by Klebanov and Maldacena [9]. Work is in progress regarding the extension of these ideas to more general supergravity backgrounds, where one finds the interesting result that the presence of NS-NS or R-R fields can cause a probe D-brane or F-string to become unstable at radii smaller than some critical value. We hope to report on this and related issues in the near future.

Note added. While this paper was being written, the three interesting works [51, 52, 53] appeared, which make remarks related to our section 4. In particular, our analysis of the potential for the long fundamental string is S-dual to the discussion of a D1-brane probe in the supergravity background dual to NCYM [54, 55] that is the subject of [53, section 3.2]. From our perspective the latter background represents the fields set up by a large number of D3-branes in Wound D-string theory [10, 11], and the results of [53] indicate that a wound D-string moving in this background sees an asymptotically flat space.

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